CHAPTER 9

CONSOLIDATION OF SOIL

دراسة إنضغاط التربة تحت تأثير الاحمال ركن أساسى من أركان ميكانيكا التربة. فى التربة غير مشبعة يكون الانضغاط أساسا بسبب نقص حجم الفراغات الهوائية ، أما فى التربة المشبعة فإن الأنضغاط يكون بسبب خروج جزء من المياه المحتوية فى الفراغات وهذا يسبب تقارب حبيبات التربة من بعضها ، ويسمى "تدعيم" ، أى أن التربة تصبح أكثر مقاومة - أو مدعمة.

ومن الطبيعى أن يعتمد المعدل الزمنى للتدعيم على نفاذية التربة. فبينما بستغرق خروج المياة من التربة الرملية وتدعيمها ثوانى أو وقت صغير جدا ، فانه يستغرق فى التربة الطينية وقت طويل وذلك للصغر المتناهى للفراغات او الممرات التى تسلكها المياه للخروج من التربة.

الواقع أن مسمى علم "ميكانيكا التربة" أطلقه العالم ترزاكى (١٨٨٣-١٩٦٣) أثر نشره لنظريته الشهيرة "نظرية التدعيم أحادى الأتجاه" “**Theory of one–dimensional consolidation**” عام ١٩٢٣ . وكان اهتمام المهندسين المشتغلين بميكانيكا التربة فى البداية مكرسا لدراسة الهبوط نتيجة الأنضغاط بالتدعيم.

ومع إنشاء معمل الأساسات فى كلية الهندسة – جامعة القاهرة عام ١٩٢٣ بدأ اجراء التجارب المعملية على عينات التربة الطينية المصرية لمعرفة خصائصها ، وفى نفس الوقت قياس هبوط أساسات المنشآت العامة كالكبارى والمبانى الإدارية.

The volume of a soil mass is decreased under stress. This decrease is known as compression, and the capacity of soil to decrease in volume under stress is know as compressibility. A soil considered as a skeleton of solid particles enclosing voids which may be filled with air or water.

9.1 Compressibility

Compression of soil may be due to:

(a) Decrease in volume of solid particles.
(b) Decrease in volume of voids.

Under the loads usually applied to soil mass, the decrease in the volume of solid particles is very small and it is reasonable to assume that the decrease in the volume of voids. If the voids are filled with air, compression will occur rapidly, since air is compressible and can escape easily from the voids. On the other hand, if the voids are filled with water, decrease in volume can be only take place when the water is expelled out (تُطرد) of the voids (assuming water is incompressible). In partly saturated soils, compression is accompanied by expulsion and compression of air and its partial dissolution (ذوبان) in water. Water may be also be expelled depending on the degree of saturation. It is important to note that while the decrease in volume of soil can be affected by vibration or mechanical means, such as tamping (طَرْق) or compaction, the term compressibility is used to indicate the capacity of soil to decrease in volume under sustained (مستمر) compressive stresses only.

The compressibility of a soil mass depends on the structural arrangement of particles, and in fine grained soils on the degree to which adjacent particles are bonded together. A loose soil is more compressible than a dense one. Also a soil composed predominantly (سائد) of flat particles is more compressible than that containing mostly spherical particles.

9.2 Compression and consolidation of laterally confined soil

انضغاط التربة المحصورة أفقيا

If a laterally confined soil specimen fitted with porous stones at top and bottom is subjected to a static stress increment ($\Delta\sigma$), compression of the soil will take place resulting in a decrease of the specimen thickness. As mentioned earlier, if the voids are filled with air, compression will occur rapidly. If, however, the voids are filled with water, the stress increment will be carried by the pore water at the instant (لحظة) of application of the stress causing an access (زيادة) in the pore water stress :

$$\Delta u_i = \Delta\sigma \qquad \text{.. .. .. (9.1)}$$

Since water is allowed to escape through the top and bottom of the sample, the excess pore stress will decrease and some of the stress increment will be carried by the soil particles such that at any time (t):

$$\Delta\sigma = (\Delta u)_t + (\Delta\sigma')_t \qquad \text{.. .. .. (9.2)}$$

where:

$(\Delta u)_t$ = excess pore water stress at time (t)

$(\Delta\sigma')_t$ = increase in the effective stress at time (t)

During the process of stress transfer (انتقال) from water to soil particles, the soil mass decreases in volume, the volume reduction being equal to volume of water drained out. This phenomenon of gradual compression due to expulsion of water from soil voids accompanied by transfer of stress from pore water to soil particles caused by application of sustained external stress is known as **primary consolidation (التدعيم الأساسى)**.

The rate of compression or consolidation of a saturated soil is governed by the rate at which pore water escapes, and hence it is directly related to soil permeability. Coarse grained soils are relatively pervious and their consolidation is assumed to be completed as rapidly as loads can ordinarily be applied in the field.

عند اللحظة الاولى من وقوع جهد اضافى على تربة طينية مشبعة لا تستجيب (لا تنفعل) التربة بهذا الجهد وتكون الحبيبات فى نفس ترتيبها فبل وقوع الحمل. وبتعبير آخر فان الجهد الفعال يكون مساويا بالكامل للجهد الاضافى. ومع مرور الوقت تتصرف المياه تدريجيا من بين حبيبات التربة فى المسار الأقرب لخارج كتلة التربة المتأثرة ويؤدى ذلك الى تقارب الحبيبات , او بتعبير آخر انتقال جزء من الجهد الاضافى الى الحبيبات , اى زيادة قيمة الجهد الفعال , ويتبع ذلك ايضا النقص فى قيمة الجهد المتعادل الذى تتحمله المياه البينية. وتستمر هذه العملية حتى يتلاشى الجهد الاضافى بالمياه البينية وينتقل عبء الجهد الاضافى بالكامل الى حبيبات ويتمثل ذلك فى تقارب حبيبات التربة اى انضغاط التربة بالقدر الذى يتناسب مع تركيبها وقيمة الجهد. ويسمى التدعيم الذى يتم الخروج جزء من المياه البينية من التربة بالتدعيم الاساسى .

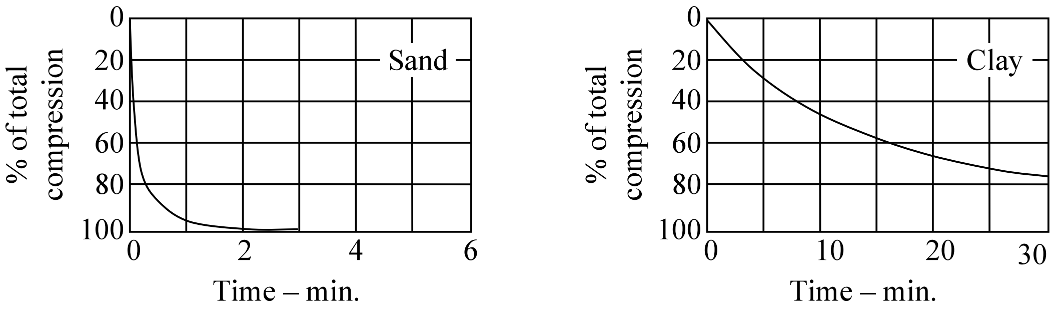
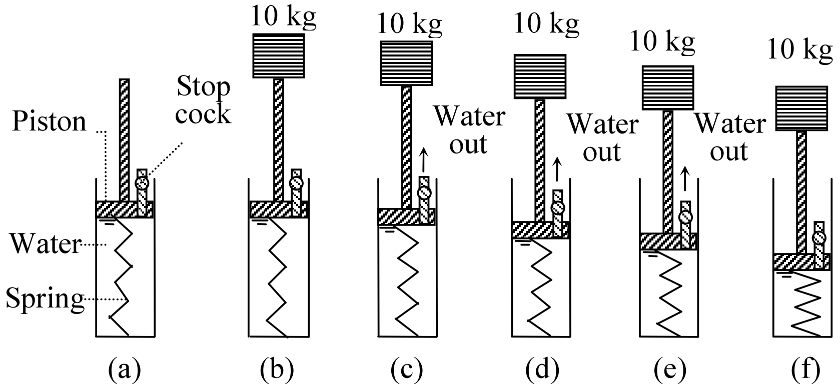


Fig. 9.1 Time-compression relation

Fig. 9.1 shows a time-compression relationship for a load increment on two types of soil, sand and clay. The curve for sand shows that the compression is almost immediate. For design purposes, the compression or settlement under a load increment in sandy soils is considered immediate and not affected by time. Fine grained soils, such as clay, possess low permeability and therefore require long time for consolidation. Fig. 9.1 shows a time-compression relationship for a load increment on both sand and clay in the one-dimensional consolidation test.

The process of consolidation may be better explained with reference to a mechanical model. Consider a spring immersed in a cylinder filled with water as shown in Fig. 9.2.



	(b)	(c)	(d)	(e)	(f)
Spring load (kg)	0	2.5	5	7.5	10
Water load (kg	10	7.5	5	2.5	0
Consolidation (%)	0	25	50	75	100

Fig. 9.2 Mechanical modelling of the consolidation process

The cylinder is fitted with a tight piston which contains a stop cock. The stopcock is closed and a weight 10 kg is placed on the piston. Since in this case (b) the water can not escape, the piston will remain in its position, and the spring will not decrease in length, i.e. no load is carried by the spring, and all the external load is carried by the water, causing an excess water stress. If the valve is opened as in case (c), water will escape due to its high stress the flow of water will continue until the excess water stress is equal to zero. During this process, the volume of water will decrease causing the piston to move downwards. The spring will consequently decrease in length, i.e. there will be a gradual increase of the load carried by the spring. Cases (c) to (f) represent the conditions after the stopcock is opened. The length of time required for the spring to reach the final length and carry all the external load depends on the rate of water dissipation, and hence the size of the stop cock opening.

In addition, if the spring is more compressible, more water would escape and consequently a longer time would be required for the spring to reach its final length. In the above mechanical model, the spring represents the compressible soil skeleton of a mass of saturated soil, and the water in the cylinder represents the water in the voids of the soil (pore water). The stopcock opening represents the compressibility of the required for consolidation increase with both the increase of soil's compressibility and decease of its permeability.

9.3 Compression characteristics (خصائص الانضغاط)

Consider a sample of clay having the consistency of slurry (روبة) and subjected to consolidation in the vertical direction only, i.e. one dimensional consolidation. At the start, the void ratio of the sample is (e_o). Then, the sample is subjected to a stress of 0.25 kg/cm^2. Consolidation will start and the void ratio will decrease gradually until at the end of the consolidation stage a void ratio of ($e_{0.25} < e_o$) is reached. At this point: $\sigma' = 0.25$ kg/cm^2.

If an increment of stress ($\Delta\sigma$) = 0.25 kg/cm^2 is applied to the sample, and again the sample is allowed to consolidate, the effective stress will be 0.5 kg/cm^2 and ($e_{0.5} < e_{0.25} < e_0$). By applying more increments of loading, a relationship between the effective stress (σ') and the final void ratio at the end of each load increment can be obtained. Fig. 9.3 (a) shows such relationship for remoulded clay (curve AE) and sand (curve LM).

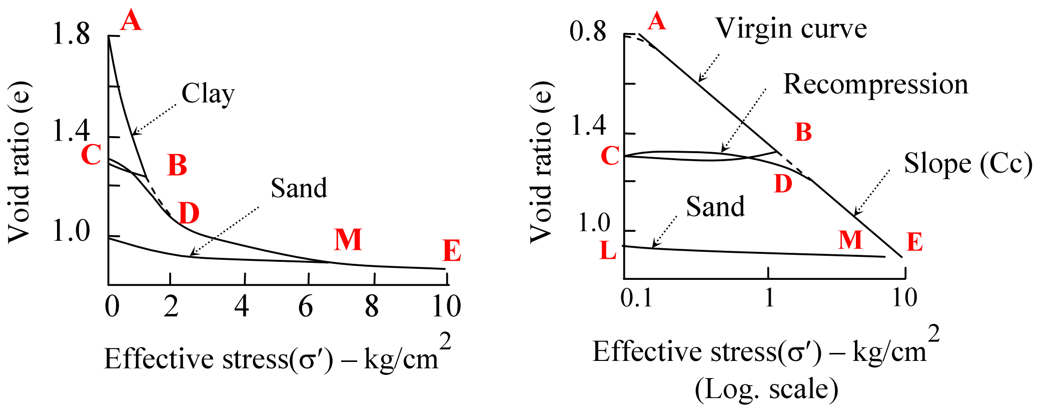


Fig. 9.3 Effective stress – void ratio relation

If at any stage of the test, such as point (B), the stress is reduced to zero and the specimen is allowed to take up water and expand, the expansion, or swelling curve (BC) is obtained. After expansion, the soil does not return to its original void ratio (e_o). This indicates a permanent change in volume, mainly due to the irreversible orientation undergone by the soil particles during compression. When the specimen is reloaded from zero to stress intensity of maximum previous compression (i.e. σ′ at B), the recompression curve (CD) is obtained. The reloaded specimen is at slightly lower void ratio than that obtained during initial compression, ($e_D < e_B$). When the stress is future increased beyond (D), the curve follows the initial curve (AB). Curve (AE) is called the virgin compression curve since at any stage in that curve the applied stress is always higher than that experienced by the soil at any period before.

9.4 Coefficient of compressibility (a_v) معامل الانضغاط

This is the ratio of the change in void ratio to the corresponding change in stress. It is equal to the numerical value of the slope of the (e - σ′) curve.

$$a_V = \frac{\Delta e}{\Delta \sigma'} \qquad \text{.. .. .. (9.3)}$$

As presented in Fig. 9.4, the slope is not constant and decreases with the increase in stress. However, for a stress increment ($\Delta\sigma'$) , (a_V) can be considered constant.

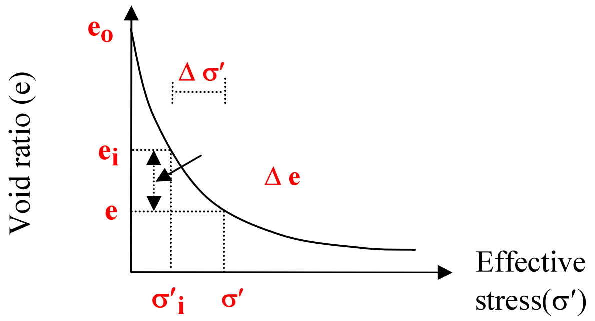


Fig. 9.4 Change of void ratio with change of

9.5 Coefficient of volume compressibility (m_v)

The volumetric strain due to a unit increase in stress is called the coefficient of volume change (m_v). Thus :

$$m_v = \frac{-\varepsilon V}{\Delta\sigma'} \qquad \text{.. .. .. (9.4)}$$

where:

$$\varepsilon_v = \text{volumetric strain}$$

$$= \frac{\Delta V}{V_i} = \frac{\Delta e}{(1+e_i)} \qquad \text{.. .. .. (9.5)}$$

$$\therefore m_v = \frac{-\Delta e}{(1+e_i)} \cdot \frac{1}{\Delta\sigma'} \qquad \text{.. .. .. (9.6)}$$

or:

$$m_v = \frac{a_v}{(1+e_i)} \qquad \text{.. .. .. (9.7)}$$

For one-dimensional consolidation, i.e. constant, area, $\varepsilon_v = \varepsilon_a$, where ε_a is the axial strain. Thus :

$$m_v = \frac{-\varepsilon a}{\Delta\sigma'} = \frac{-\Delta H}{H} \cdot \frac{1}{\Delta\sigma'} \qquad \text{.. .. .. (9.8)}$$

where:

ΔH = change of height due to consolidation under ($\Delta\sigma'$)
H = original height.

9.6 Relation between void ratio and logarithm of effective stress

If the stress (σ') is plotted as abscissa on a logarithmic scale and the void ratio as ordinate, the virgin curve become a straight line, as shown in Fig. 9.5. The empirical equation of the line is :

$$e = e_i - c_c \log_{10} \frac{\sigma'}{\sigma'_i} \qquad \text{.. .. .. (9.9)}$$

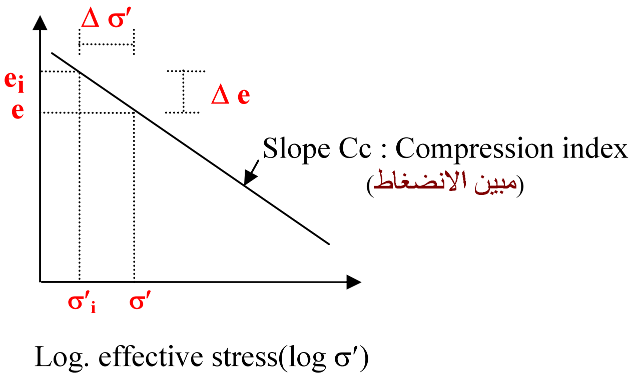


Fig 9.5 The (e – log σ′) relation

where C_c is a dimensionless value representing the slope of the linear portion of the stress-void ratio curve on a semi-log plot, and is termed the compression index.

The expansion or swelling curve can also be considered a straight line on a semi – log plot and the numerical value of its slope is termed **expansion index** (مبين الانتفاش) (C_e), Fig. 9.6:

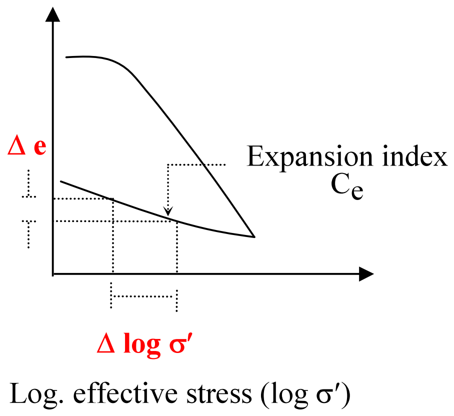


Fig. 9.6 Calculation of the expansion index

9.7 Terzaghi's Theory of Consolidation

تعنى نظرية ترزاكى بحساب قيمة ضغط المياه البينية عند نقطة على عمق معين من التربة بعد زمن معين نتيجة تعرض التربة لجهد اضافى . ويتبع ذلك حساب الانضغاط بالتدعيم والهبوط المقابل لسطح الارض نتيجة الانتقال التدريجى للجهد الاضافى الى حبيبات كجهد فعال .

يفترض ترازكى فى نظريته ان التربة متجانسة ومشبعة تماما ويهمل الانضغاط بالغ الصغر الذى يحدث للحبيبات والمياه ، كما يفترض امكانية تطبيق نظرية دارسى وثبات قيمة معامل نفاذية التربة ، وان التدعيم الناتج سيتم فى الاتجاه الرأسى فقط .

Terzaghi (1923) presented a mathematical theory for one dimensional consolidation. The main assumptions used in this theory are :

(a) The soil is homogenous and fully saturated.
(b) The soil particles and water in voids are incompressible.
(c) Darcy 's law of flow of water through soil is valid.
(d) The coefficient of permeability is constant.
(e) The soil is laterally confined so that the resulting compression is one dimensional.
(f) Excess pore water drains out only in the vertical direction.
(g) The effective stress – void ratio relationship is linear.

The rate of volume change of a soil element dx, dy, dz due to flow of water in the voids is:

$$\frac{\partial V}{\partial t} = \left(k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{a z^2} \right) dxdy.dz \qquad .. .. (9.10)$$

For steady flow, i.e. no volume change due to the flow of water, the left hand side of Equ.(9.10) is equal to zero. Thus in the case of two dimensional flow in isotropic material, we get Laplace equation :

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

For transient flow the left hand side of Equ. (9.10) is not equal to zero. The change in the volume of the soil element is due to escape of pore water when the element is subjected to sustained stress. In one dimensional consolidation, water flows in one direction only, and Equ.(9.10) reduces to :

$$\frac{\partial V}{\partial t} = k_z \frac{\partial^2 h}{\partial z^2} dx.dy.dz \qquad \text{.. .. .. (9.11)}$$

Since soil particles and pore water are assumed incompressible, the change of volume of the element will result from the change in the volume of voids (Vv). Then :

$$V_V = \frac{e}{e+1} dx.dy.dz$$

$$\frac{\partial V}{\partial t} \frac{\partial V_V}{\partial t} = \frac{\partial e}{\partial t} . \frac{dx.dy.dz}{1+e} \qquad \text{.. .. .. (9.12)}$$

Note that (dx.dy.dz) / (1+e) is constant since it is equal to the volume of the solids. Now, substituting for $\partial V / \partial t$ from Equ. (9.12) into Equ.(9.1), then :

$$\frac{\partial e}{\partial t} . \frac{dx .dy .dz}{1+e} = k_z \frac{\partial^2 h}{\partial z^2} dx.dy.dz$$

$$\frac{1}{1+e} . \frac{\partial e}{\partial t} = k_z \frac{\partial^2 h}{\partial z^2} \qquad \text{.. .. .. (9.13)}$$

The excess hydrostatic pressure (u) which cause the water to flow is given by :

$$u = \gamma_w \cdot h \qquad \text{.. .. .. (9.14)}$$

Substituting for (h) from Equ. (9.14) into Equ. (9.13) :

$$\frac{1}{1+e} . \frac{\partial e}{\partial t} = \frac{k_z}{\gamma_w} \frac{\partial^2 u}{\partial z^2} \qquad \text{.. .. .. (9.15)}$$

When the applied load is sustained, the consolidation which occurs results in gradual decrease in (u) and a corresponding increase in the effective (σ').
Thus :

$$\frac{\partial u}{\partial t} = -\frac{\partial \sigma'}{\partial t}$$

It has been shown that : $a_V = -\frac{\Delta e}{\Delta \sigma'}$, therefore:

$$a_V \frac{\partial u}{\partial t} = \frac{\partial e}{\partial t} \qquad \text{.. .. .. (9.1٦)}$$

Substituting from Equ.(9.16) into Equ.(9.15):

$$\frac{a_V}{1+e} \cdot \frac{\partial u}{\partial t} = \frac{k_Z}{\gamma_W} \cdot \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial u}{\partial t} = \frac{k_Z}{\gamma_W} \cdot \frac{1+e}{\partial_V} \cdot \frac{\partial^2 u}{\partial z^2}$$

$$\therefore \quad \boxed{\frac{\partial u}{\partial t} = C_V \frac{\partial^2 u}{\partial z^2}} \qquad \text{.. .. .. (9.17)}$$

where : $C_V = \frac{k_Z}{\gamma_W} \cdot \frac{1+e}{a_V} = \frac{k_Z}{\gamma_W} \cdot \frac{1}{m_V}$

المعنى الطبيعى للمعادلة التفاضلية المبينة اعلاه ان ضغط المياه البينية يتغير حسب الزمن المنصرم منذ بدء التحميل , وعمق النقطة التى يتم حساب الضغط فيها ، كما انه يتوقف على ثوابت وهى معامل النفاذية ومعامل الانضغاط الحجمى للتربة . وقد تم حل هذه المعادلة بالطرق الرياضية التقليدية وأدخل فى الحل متغير سمى "معامل الزمن" . هذا المتغير يتوقف على درجة الانضغاط بالتدعيم وتوزيع ضغط المياه البينى على طبقة التربة قبل وقوع الجهد الاضافى عليها ، وليس على خواص التربة .

C_V is called the **coefficient of consolidation** (معامل التدعيم), and its value is governed by both the permeability and the compressibility of the soil. The last equation can be put in the from :

$$k_V = C_V \cdot m_V \cdot \gamma_W \qquad \text{.. .. .. (9.1٨)}$$

Equ. (9.17) is the fundamental differential equation for the one dimensional theory of consolidation. A solution for this equation for the simplified condition shown in Fig. 9.7 is given as follows :

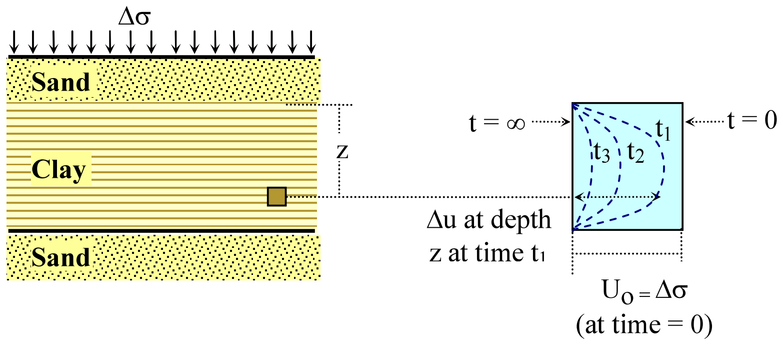


Fig. 9.7 Pore water pressure dissipation during consolidation

$$u_z = \frac{4}{\pi}\Delta\sigma' \sum_{N=0}^{N=\infty} \frac{1}{2N+1}\sin\left[\frac{(2N+1)\pi z}{H}\right] e^{\frac{-(2N+1)^2\pi^2}{4}\cdot T_v} \qquad \text{.. .. .. (9.19)}$$

where : u_z =excess pore pressure at depth (z) at any time (t).
N = positive integer.
T_v = dimensionless parameter.

T_v is called the **time factor** (معامل الزمن), defined by:

$$T_v = \frac{C_v \cdot t}{d^2} \qquad \text{.. .. .. (9.20)}$$

where : d = length of the drainage path (مسار الصرف).

The value of d = H/2 where a soil layer of thickness H is drained from both top and bottom ends, and d = H where a soil layer of thickness H is drained from one side only.

At any given time t the distribution of the excess pore water pressure can be calculated using Equ. 9.19, and plotted as shown in Fig. 9.7. The resulting curves are called isochrones.

9.8 Degree of consolidation U_z at depth z and time t

The degree of consolidation (درجة التدعيم) U_z is defined as the ratio of the vertical settlement of the soil element, at depth z and time t, to its final settlement, i.e.:

$$U_Z = \frac{s_Z}{s_\infty}$$

$$= \frac{m_V \cdot (\Delta\sigma')_t \cdot H}{m_V \cdot (\Delta\sigma')_\infty \cdot H} = \frac{(\Delta\sigma')_t}{(\Delta\sigma')_\infty} = \frac{\Delta\sigma - (\Delta u)_t}{\Delta\sigma} \qquad \text{.. .. .. (9.2١)}$$

For the case shown in Fig . 9.7, $\Delta u_o = \Delta \sigma$, then :

$$U_Z = \frac{(\Delta u)_o - (\Delta u)_t}{(\Delta u)_o} = 1 - \frac{(\Delta u)_t}{(\Delta u)_o} \qquad \text{.. .. .. (9.2٢)}$$

Thus the distribution of U_Z across the depth is similar to the distribution of the excess pore pressure U_Z , with the values of U_Z calculated from Equ. 9.22.

9.9 Average degree of consolidation (U)

The degree of consolidation at any depth is normally not of practical interest. It is more convenient to work in terms of the average of consolidation (U) when determining the change in the thickness of the soil layer as a whole. (U) is defined as :

$$U\% = \frac{s_t \text{ of the layer}}{s_\infty \text{ of the layer}} \times 100 \qquad \text{.. .. .. (9.23)}$$

At any time t , the change in the thickness of a soil element is:

$$\Delta H = m_V \cdot \Delta\sigma' \cdot \Delta z$$

Integrating between the limits 0 and H, the settlement $(s)_t$ of the full thickness of the soil layer at time t is obtained. Then :

$$s_t = \int_0^H m_V (\Delta\sigma - u_Z) dz$$

$$= m_V \cdot \left(\Delta\sigma \cdot H - \int_0^H u_Z \cdot dz \right)$$

Substituting for U_z from Equ. (9.19), and integrating:

$$S_t = m_v \; \Delta\sigma \; H \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{N=\infty} \left(\frac{1}{(2N+1)^2} \right) e^{\frac{-(2N+1)^2 \pi^2}{4} . T_v} \right]$$

At time t = ∞, the final settlement s_∞ is given by: $S_\infty = m_v \; \Delta\sigma \; H$

$$\text{Thus : } \; U\,\% = \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{N=\infty} \left(\frac{1}{(2N+1)^2} \right) e^{\frac{-(2N+1)^2 \pi^2}{4} . Tv} \right] \times 100 \quad \text{... (9.24)}$$

The relationship between U and T_v expressed in Equ. (9.24) is given for different cases, as shown in Fig. 9.8, by different corresponding curves.

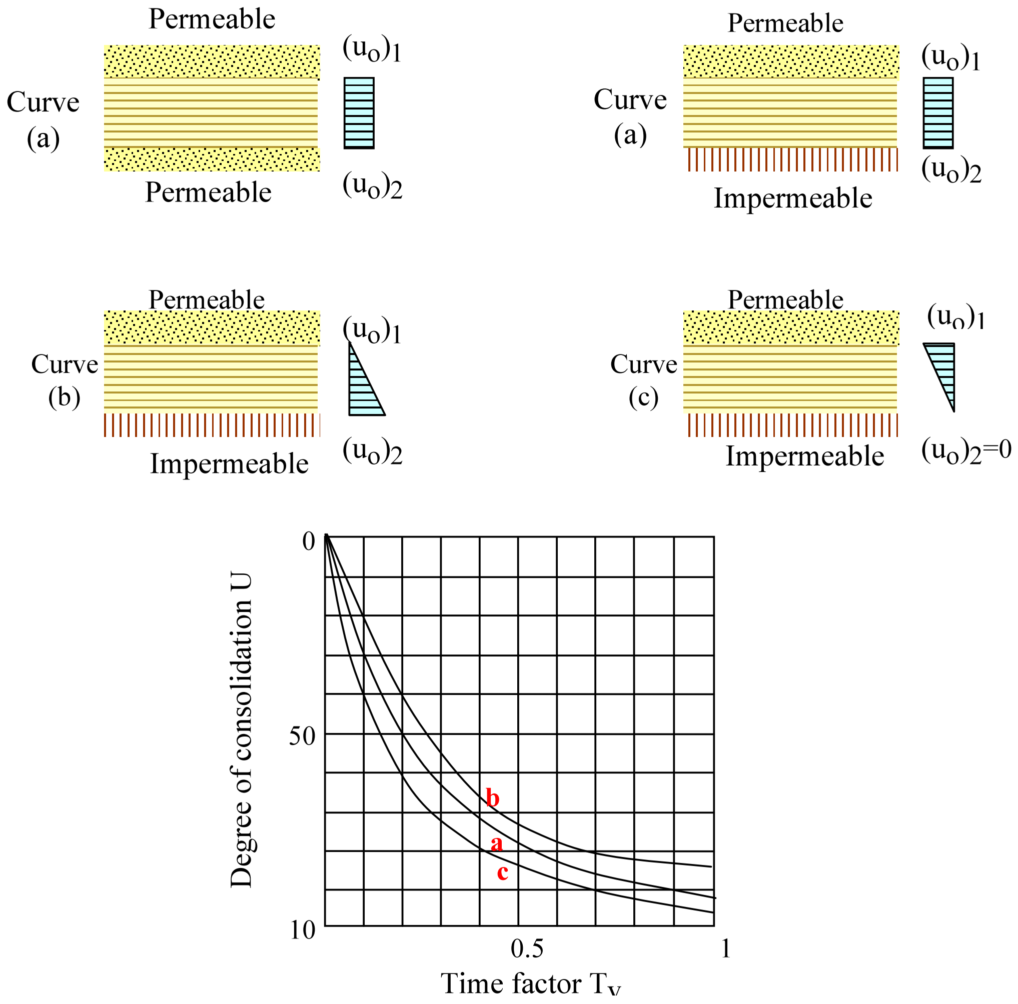


Fig. 9.8 Solution of one-dimensional consolidation

The relationship given by curve (a) can be represented by the following empirical expressions:

$$\text{For} \quad U < 52.6\,\% \quad : \; T_V = (\pi/4).\,U^2 \qquad \ldots\ldots\ldots\ldots (9.25)$$

$$\text{For} \quad U < 52.6\,\% \quad : T_V = -\,0.9332 \log 10\,(1 - U) - 0.0851 \qquad \ldots\ldots (9.26)$$

Referring to Fig. 9.8, it should be noted that the area under an isochrone at a certain time (t) represents the amount of consolidation still to be undergone by the soil layer at this particular time. The total consolidation is represented by the total area of the initial excess pore water stress distribution diagram.

Therefore, the average degree of consolidation (U) at time (t) can be determined from :

$$U\% = \left(1 - \frac{\text{Area under isochrone at time (t)}}{\text{Area of initial pore water pressure diagram}}\right) \times 100 \qquad \ldots\ldots (9.27)$$

9.10 Consolidation test إختبار التدعيم

إختبار التدعيم من أهم الاختبلرات الرئيسية فى مجال ميكانيكا التربة ومنه تستخلص المعاملات التى يمكن بواسطتها حساب هبوط التربة الطينية نتيجة تدعيمها بجهد اضافى معين ناشىء عن حمل مبنى او خلافه , وكذلك حساب معدل زيادة هذا الهبوط مع الزمن. ويجرى الاختيار على عينة غير مقلقلة تمثل طبقة التربة المعرضة للتدعيم .

In order to determine the compression characteristics of a soil, a consolidation test is carried out in an apparatus called oedometer (إيدومتر), Fig. 9.9.

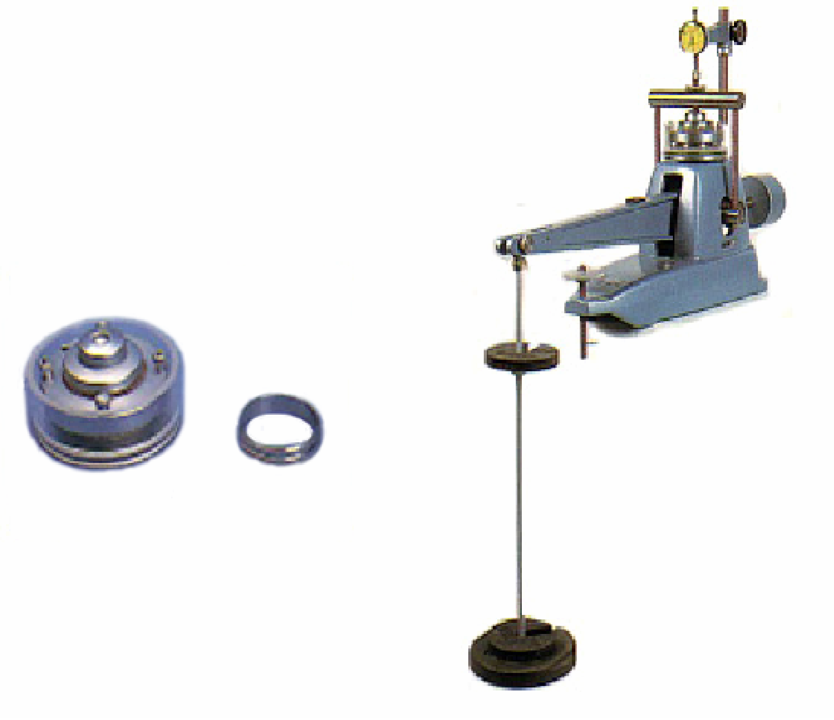


Fig. 9.9 The oedometer and loading frame

9.10.1 Procedure

The soil specimen is placed in a metal ring and porous stone discs are placed on the top and bottom of the specimen. The test specimen is allowed to consolidate under a number of increments of vertical stress, each stress being maintained constant until the compression ends (usually 24 hours). The stress usually employed are 0.25, 0.5, 1, 2, 4, 8 kg/cm^2. The readings of the vertical compression of the specimen are recorded against time (readings are usually taken at 0.5, 1, 2, 4, 8, 15, 30 min, and 1, 2, 4, 8, 16 and 14 hours).

9.10.2 Calculations

(a) Determination of the coefficient of consolidation (C_v)

(i) Taylor 's method :

This method is known as the square root of time fitting method. The compression readings are plotted against (√ t). The upper portion of the curve is theoretically a straight line which is extended upwards to determine the initial value of compression at the beginning of the consolidation process, and also extended downwards to meet the x – axis, Fig. 9.10. A line with abscissa 1.15 times that of the first straight line is drawn from which t 90 (time for 90% consolidation) and H90 (height of specimen at 90% consolidation) are obtained. The Coefficient of consolidation C_v is obtained from :

$$T_V(90) = \frac{C_V \cdot t_{90}}{(d_{90})^2} \qquad \text{.. .. .. (9.28)}$$

For the drainage conditions described for the oedometer T_v = 0.848, and $d_{90}=(H_{90})/2$.

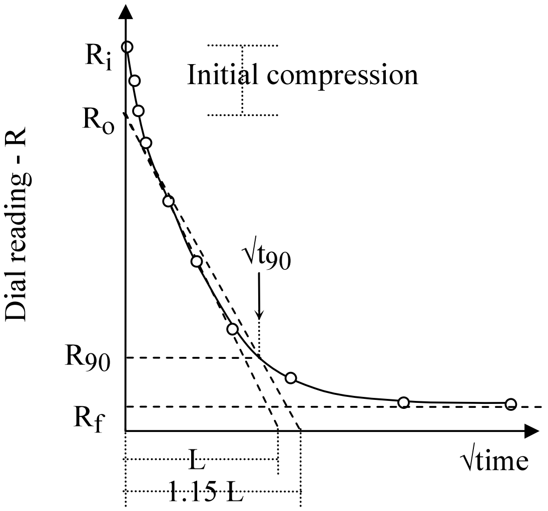


Fig. 9.10 Square root of time fitting method

(ii) **Casagrande's** method :

This method is known s the logarithm of time fitting method. The compression readings are plotted against $\log_{10}$ t. The values of H_0, H_{100} and H_{50} are determined as shown in Fig. 9.11.

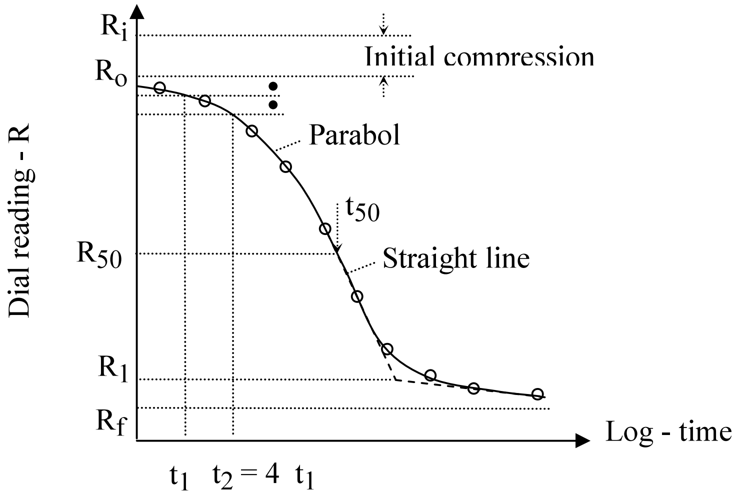


Fig. 9.11 Logarithm of time fitting method

The coefficient of consolidation C_v is determined from the following equ.:

$$T_{v(50)} = \frac{C_v \cdot t_{50}}{(d_{50})^2} \qquad \text{.. .. .. (9.29)}$$

(b) Determination of coefficient of volume change (m_v)

The coefficient of volume change is determined by knowing the void ratio at the end of each stress increment. Let H_0 be the initial height of the specimen, and cross sectional area A . The final height of the specimen at the end of the test is determined from :

$$H_f = H_o - (\Delta H)_f$$

where $(\Delta H)_f$ is the total change in the height of the specimen measured by the dial gauge, at the end of the test. Now, by determining the final water content of the specimen (w_f) the final void ratio is determined by :

$$e_f = G_s \cdot w_f$$

The void ratio at any stress increment can be calculated by knowing the difference in specimen height $\Delta H = H - H_f$ and applying the relation :

$$\frac{\Delta H}{H_f} = \frac{\Delta e}{1 + e_f} = \frac{e - e_f}{1 + e_f} \qquad \text{.. .. .. (9.30)}$$

Calculations are processed backwards, i.e. from last to first stress increment, and all the void ratio at all the stress increments are obtained. The results are tabulated as follows:

Applied stress σ'	Final dial reading	Δ H mm	H mm	Δ e	e

Now, m_v can be calculated from any stress increment from the relations:

$$m_v = \frac{\Delta e}{1 + e_i} \cdot \frac{1}{\sigma' - \sigma'_i} \qquad \text{or:}$$

$$m_v = \frac{\Delta H}{H_i} \cdot \frac{1}{\sigma' - \sigma_i'} \qquad \text{.. .. .. (9.31)}$$

(c) Determination of the coefficient of permeability (k)

The coefficient of permeability (k) can be calculated from the relation:

$$k = C_v \, . \, m_v \, . \, \gamma_w$$

(d) Pre-consolidation stress ($\sigma`_c$) إجهاد التدعيم المسبق

When a soil is removed from the ground, loads are reduced, therefore when the sample is tested in the consolidation test, the first part of the e-log σ curve actually corresponds to recompression. It is therefore important to determine the in–situ previous consolidation stress from the laboratory e-log σ` relationship. An empirical method is as follows:

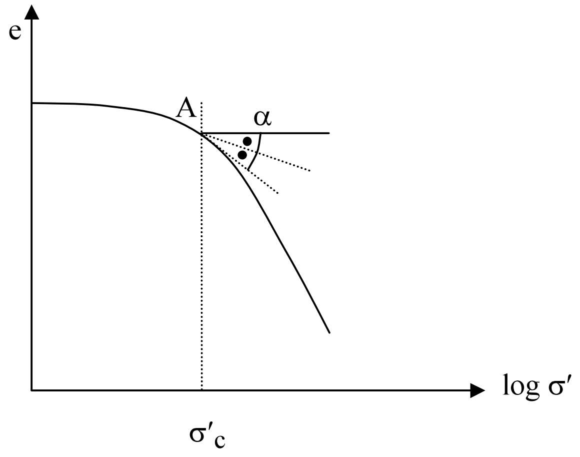


Fig. 9.12 Determination of the preconsolidation stress

(a) From the e–log plot determine the point of minimum radius (A), Fig. 9.12.

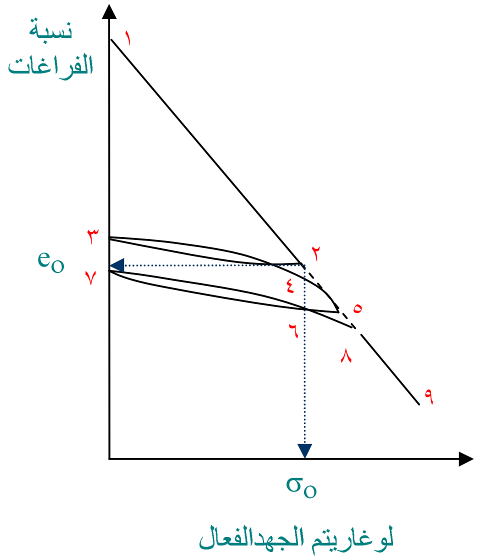
(b) From (A) two lines are drawn one tangent to the curve and the other parallel to the log σ' axis.

(c) The angle (α) between the two lines is bisected.

(d) The intersection of the bisector and the extension of the straight portion of the plot is determined. The abscissa of the point of intersection gives the preconsolidation stress (σ'_c).

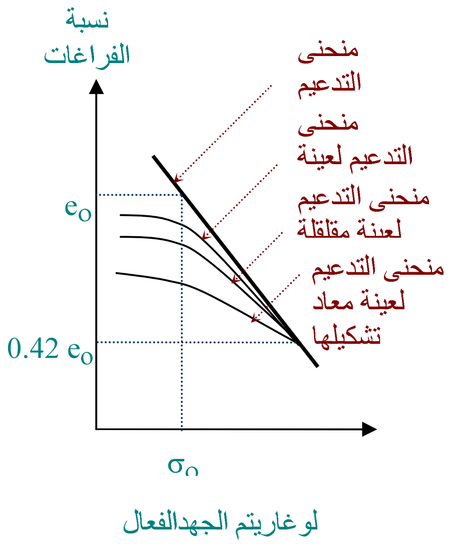
9.11 Field consolidation curve

فى بدء الترسيب تكون التربة الطينية فى حالة سيولة ونسبة الفراغات عالية جدا ، وهذا ممثل بالنقطة ١ فى الشكل. مع تتابع الترسيب يزداد الجهد على التربة السفلية تدريجيا وتتدعم الى ان تنتهى "تاريخيا" عملية الترسيب وتستقر التربة بمحتوى رطوبة ونسبة فراغات معينة – نقطة ٢.



اذا لم تتعرض التربة للنحر او ازالة الطبقات العلوية منها او الانشاء فيقال عن تاريخ الاجهادات على التربة ان التربة مدعمة تدعيما عاديا . عند استخراج عينة من هذه التربة فان الجهد الواقع عليها (σ_o) سنخفض الى الصفر مما يجعلها تنتفش قليلا – نقطة ٣ – وعند تحميا العينة فى اختبار التدعيم فان العلاقة بين الجهد ونسبة الفراغات ستأخذ مسارا تتغير فيه نسبة الفراغات قليلا مع زيادة الجهد حتى وصول الجهد الى قيمة الفعال الواقع على العينة فى الطبيعة , حيث يبدأ المنحنى فى الانثناء بحده – نقطة٤ – لكى تستمر العلاقة بين لوغاريتم الجهد ونسبة الفراغات نفس المسار الاصلى ١-٢ . اذن .. اذا كانت التربة عادية التدعيم فان انثناؤ المنحنى الذى يمثل العلاقة بين لوغاريتم الجهد ونسبة الفراغات سيكون متوافقا مع الجهد من وزن طبقات التربة الحالية (σ_o) .

قد تتعرض التربة لمتغيرات فى الجهد الواقع عليها بعد اتمام عملية الترسيب نتيجة عوامل التجوية او تغير منسوب المياه الجوفية او اقامة منشأ ثم هدمه وفى هذه الحالة سيكون لتاريخ التدعيم امتداد , اذ ان العلاقة بين لوغاريتم الجهد ونسبة الفراغات ستستمر فى مسارها حتى اقصى جهد تتعرض له التربة - نقطة ٥ – وعند نقص هذا الجهد تأخذ العلاقة مسارا انتفاشيا لتصل الى نقطة ٦ . عند استخراج عينة من هذه التربة فان الجهد الواقع عليها (σ_o) سنخفض الى الصفر مما يجعلها تنتفش قليلا – نقطة ٧ – وعند تحميل العينة فى اختبار التدعيم فان العلاقة بين الجهد ونسبة الفراغات ستأخذ مسارا تتغير فيه نسبة الفراغات قليلا مع زيادة الجهد حتى وصول الجهد الى اقصى قيمة سبق الوصول اليها (وهو اعلى من الجهد الواقع على العينة فى



الطبيعة وقت استخراجها (ممثلة فى النقطة ٨ , بعدها يتقارب المسار ليأخذ نفس المسار الحقلى الاصلى وهو امتداد الخط ١ – ٢ . وبذلك .. فانه اذا كانت التربة – كتاريخ اجهادات – سابقة التدعيم فان انثاء المنحنى الذى يمثل العلاقة بين الجهد ونسبة الفراغات سيكون عند جهد اعلى من الجهد من وزن طبقات التربة الحالية .

وهناك ايضا حالة الطين غير كامل التدعيم , وهى الحالة التى يكون فيها الجهد الواقع على التربة حاليا او جزء منه بسبب وزن ردم حديث بفعل الطبيعة او الانسان ويؤدى الى تدعيم التربة وهذا التدعيم لا يزال جاريا وقت استخراج عينة التربة منها .

عموما .. فى جميع الاحوال يجب ان يحسب الهبوط الناشئ عن التدعيم من المسار الاصلى الحقلى للعلاقة بين الجهد ونسبة الفراغات , وهو المسار ١ – ٢- ٣ – ٤ – ٥ – ٩ , كما فى الشكل .

فى الصفحات التالية شرح لكيفية استنتاج منحنى التدعيم الحقلى للحالات الثلاثة من تاريخ الجهد على التربة وهى حالات ان يكون الطين العادى او مسبق او غير مكتمل التدعيم . هذه الطرق تعتمد على اساسا على ابحاث شيمرتمان (١٩٥٥) الذى استنتج من عديد من التجارب ان منحنيات التدعيم الحقلية ولعينات التربة غير المقلقلة والمقلقلة والمعاد تشكيلها بنفس محتوى الرطوبة تتقابل جميعها عند نسبة فراغات حوالى ٤٢ % من نسبة الفراغات الحقلية .

Consider a soil element at depth (z). The element is subjected to an effective stress σ′ due to weight of soil above it. This stress, due to the overlying soil, is called **overburden stress**. Comparing between the overburden stress and the pre-consolidation stress, the stress history of soil can be visualized, and hence the field consolidation curve can be obtained. Three cases are to be considered:

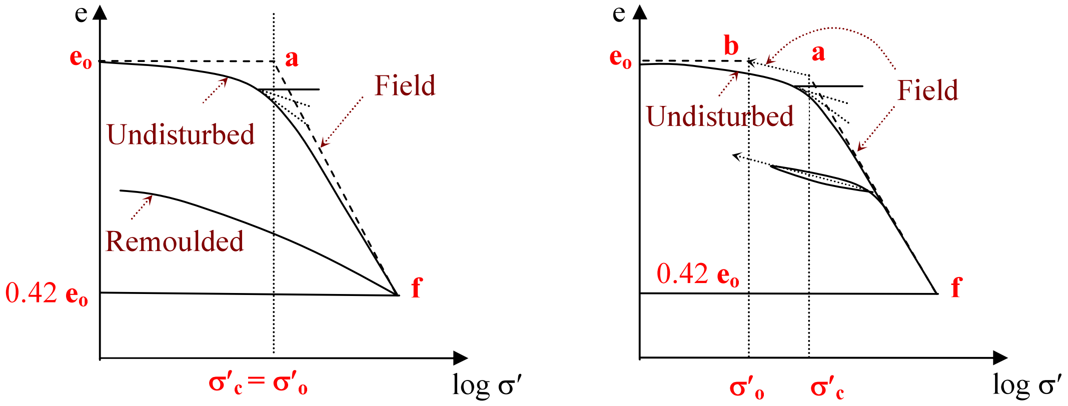
9.11.1 Normally consolidation clay الطين عادى التدعيم

In this case $\sigma_c' = \sigma_o'$. It was experimentally found that sample disturbance and remoulding of soil sample will cause decrease in the slope of the straight position of the e – log σ ′ curve. However, it was also found that the field curve, undisturbed laboratory curve, and remoulding laboratory curve intersect at a void ratio 0.42 e_o. Therefore, the procedure for finding the field consolidation curve is :

a. Determine on the undisturbed or remoulded curve the point corresponding to $0.42\ e_o$, Fig. 9.13, point f .

b. Determine point (a) corresponding to e_o and σ_o'.

c. Line a – f is the field consolidation line.



Normally consolidated clay

Over consolidated clay

Fig. 9.13 Determination of stress history and field consolidation curve

For normally consolidation clays the compression index Cc for the field consolidation curve can be obtained for the following empirical relationship :

$$C_c = 0.009\ (LL\ \% - 10) \qquad \text{.. .. .. (9.32)}$$

where: LL is the liquid limit of the soil.

9.11.2 Over-consolidated clay

In this case the pre-consolidation stress in higher than the overburden stress, i.e., $\sigma_c' > \sigma_o'$. This occurs in nature due to a load that was exerted on the soil for a period of time then removed by weathering or man operations. Field consolidation curve is obtained as follows, Fig. 9.13 :

a. Point b is determined, which corresponds to e_o and log σ_o'.

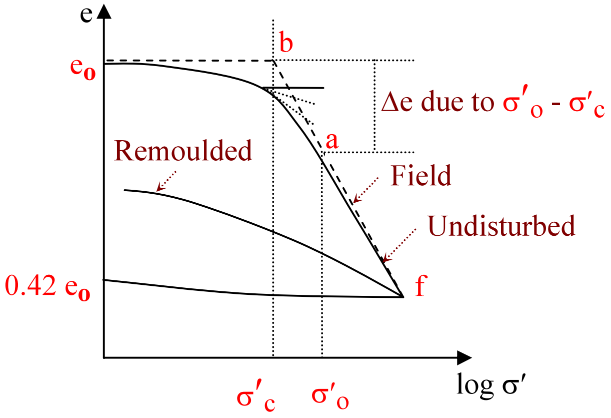
b. Point f is determined on the laboratory curve, corresponding to $0.42\ e_o$.

c. Point a is determined from the intersection of line b parallel to the average slope of the laboratory reloading curve and the vertical line corresponding to σ_c'.

d. The field compression curve is then b-a-f.

9.11.3 Under-consolidation clay

In this case the reconsolidation stress is found less than the overburden stress, i.e., $\sigma_c' < \sigma_o'$. This occurs when the element of soil not fully consolidated under the overburden stress, for example in the case of recent fill deposit. The field consolidation curve is obtained as shown in Fig. 9.14. It should be noted that when calculating the settlement in the field due to external loads, the consolidation due the difference between σ_o' and σ_c' should taken into consideration.



Under consolidated clay

Fig. 9.14 Determination of field consolidation for under consolidated clay

9.12 Consolidation Settlement الهبوط نتيجة التدعيم

If a compressible soil element of thickness Δz is subjected to an increase of the effective stress Δσ′, it will exhibit ultimate

Consolidation settlement Δs_f, where:

$$\Delta s_f = m_v \cdot \Delta\sigma' \cdot \Delta z$$

If both m_v and $\Delta\sigma'$ are constant for the whole layer of thickness (H), then:

$$s_f = m_v \cdot \Delta\sigma' \cdot H \qquad \text{... ... ...} \; (9.33)$$

If the layer is relatively thin and the stress varies linearly with depth, Equ. 9.34 may be used for calculated the final consolidation settlement. The intensity of stress at the mid–height of the layer is considered to represent a constant average stress increment for the layer and is taken as $\Delta\sigma'$. Similarly, an average value of m_v may be also assumed constant. Thus, if σ'_i and σ'_f are the initial effective stresses at the middle of the layer, mv will be the coefficient of volume change for the stress range σ'_i to σ'_f, and :

$$\Delta\sigma' = \sigma'_f - \sigma'_i$$

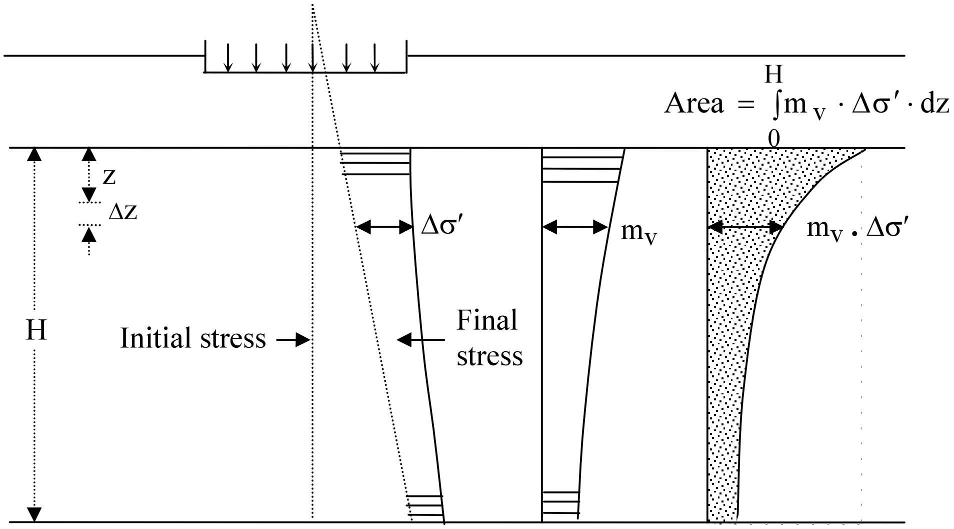
If the intensity of stress $\Delta\sigma'$ varies non – linearly with depth, m_v will also vary, and the final settlement will be obtained by integrating the following equation:

$$s_f = \int_0^H m_v \cdot \Delta\sigma' \cdot dz \qquad \text{.. .. .. (9.34)}$$

This integration can be done as follows:

a. Divide the layer into thin layers.
b. The settlement of each layer corresponding to its average $\Delta\sigma'$ and m_v, is calculated.
c. To obtain the total final settlement, all the values obtained in (b) are added.

This is presented graphically as shown in Fig. 9.15.



Area = $\int_0^H m_v \cdot \Delta\sigma' \cdot dz$
z
Δz
H
Initial stress
Δσ′
Final stress
m_v
$m_v \cdot \Delta\sigma'$

Fig. 9.15 Settlement calculation for soil layer with varying increase of stress and varying m_v

For normally consolidated clay, the final consolidation settlement may be calculated using the compression index C_c as follows:

$$\frac{s_f}{H} = \frac{\Delta e}{1+e_i}$$

$$\Delta e = \frac{C_c}{1+e_i} \log_{10} \frac{\sigma_i' + \Delta\sigma'}{\sigma_i'}$$

$$s_f = \frac{C_c}{1+e_i} \cdot H \cdot \log_{10} \frac{\sigma_i' + \Delta\sigma'}{\sigma_i'} \qquad \text{.. .. ..} \; (9.35)$$

يلاحظ فى تطبيق العلاقة (9.36) ان يكون المعامل (Cc) مناسبا لمدى الجهد من $(\sigma'i + \Delta\sigma')$. فاذا كان الطين عادى التدعيم فتقاس قيمة (Cc) من منحنى التدعيم الحقلى من تجربة التدعيم او تحسب تقديريا من العلاقة (9.33). اما اذا كان مدى الجهد فى نطلق لا يتعدى الجهد المسبق (σ_c) فان المعامل (Cc) يكون هو (C_r = reloading compression index) الذى يساوى تقريبا (C_e) كما هو معرف فى شكل (9.6). واذا كان مدى الجهد فى نطاق يتعدى الجهد المسبق (σ_c) فان الهبوط يجزأ الى قيمتين تحسب الأولى بالمعامل (C_c) لمدى الجهد من (σ'_i) الى (σ_c) والثانية بالمعامل (C_r) لمدى الجهد من (σ_c) الى ($\sigma_i' + \Delta\sigma'$) . وللحسابات التقريبية تستخدم العلاقة (9.34) وتؤخذ قيم (m_v) من الجدول الآتى:

Soil type	$m_v - cm^2/kg$
Very highly compressible	0.1
Highly compressible	0.1 – 0.02
Medium compressibility	0.02 – 0.005
Low compressibility	0.005 – 0.002
Very low compressibility	< 0.002

9.13 Secondary compression

الانضغاط الثانوى ظاهرة تحدث غالبا نتيجة تشوه هيكل التربة بدون تغير فى محتوى الرطوبة. وتعتبر معظم النظريات ان هذا الانضغاط يحدث بعد انتهاء الانضغاط او التدعيم الاساسى. فى التربة العضوية يمثل الانضغاط الثانوى نسبة عالية جدا من الانضغاط الكلى ولابد فى حسابات الهبوط من اخذة فى الاعتبار ، أما فى التربة غير العضوية فتهمل قيمته.

It has been mentioned earlier that compression of clay takes a very long time. Two factors may be responsible for this time delay:

(a) Hydrodynamic lag, or primary compression which is due basically to permeability which controls the flow of pore water.
(b) Plastic lag, or creep.

For many soils, secondary compression is neglected. For organic soils it may constitute a substantial part of the total

The following Equ. is used to calculate the secondary (Δs)

$$\Delta s = C_{\alpha} . H . \log_{10}\left\{\left(t_{life} - t_{p.c.}\right) / t_{p.c}\right\} \quad \text{.. .. .. (9.36)}$$

where:

C_{α} = slope of the tail of the (ΔH – log 10 t) relation
t_{life} = life time of the structure
$t_{p.c.}$ = time to end primary consolidation
H = height of soil layer

Typical values of are given in the following table:

Soil type	C_{α} – cm^2/kg
Organic	0.03
Normally consolidated clay	0.005 – 0.02
Over-consolidated clay	< 0.001

9.14 Examples

(1) A consolidation test was carried out on a soil specimen 19 mm high. The field bulk density and water content are 1.83 t/m3 and 33.91 %, respectively. Results of the test are as following :

$\sigma' = \text{kg/cm}^2$:	0	0.1	0.25	0.5	1	2	4	8	0
Dial reading x 0.002 mm :	0	22	70	178	342	496	648	769	625

The final water content was 32 %. Draw the e-log relationship. Calculate C_c, and m_v for stress range 0.5 to 1 kg/cm^2. Take $G_s = 2.7$.

Solution:

$$H_f = 19 - 625 \times 0.002 = 17.75 \text{ mm}$$

$$e_f = 2.7 \times 0.32 = 0.864$$

Apply the relation :

$$\frac{\Delta H}{H} = \frac{\Delta e}{1 + e}$$

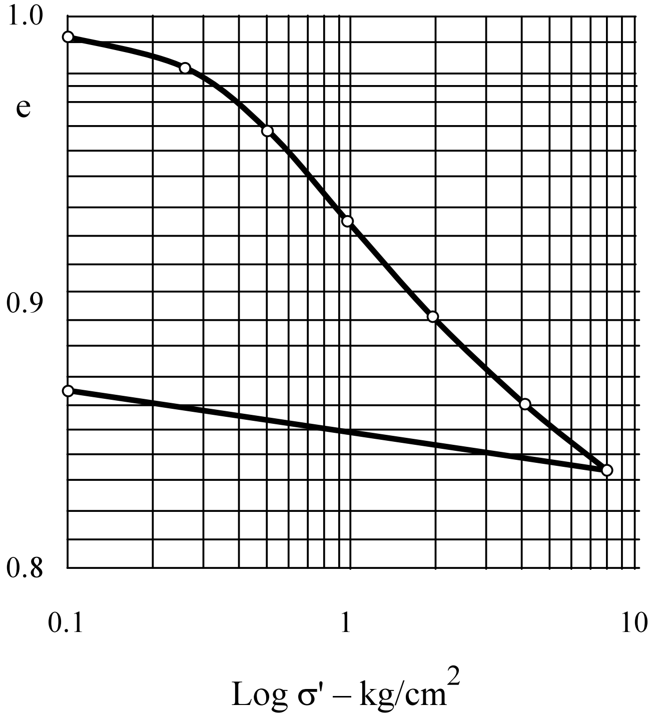
$$\frac{H - 17.75}{17.75} = \frac{e - 0.864}{1 + 0.864}$$

$$\therefore e = 0.105 \times (H - 17.75) + 0.864$$

From this relation the following table can be calculated :

σ' kg/cm^2	Dial reading x 0.002 mm	ΔH mm	H mm	Δe $e - e_f$	e
0	0	0.000	19.000	0.131	0.995
0.1	22	0.044	18.956	0.127	0.991
0.25	70	0.140	18.860	0.117	0.981
0.5	178	0.356	18.644	0.096	0.958
1	342	0.684	18.316	0.059	0.923
2	496	0.992	18.008	0.027	0.891
4	648	1.296	17.704	0-.005	0.859
8	796	1.538	17.462	-0.300	0.834
0	625	1.25	17.750	0.000	0.864

Draw e-log σ' relation:



Notice that e_o can be calculated from initial data, its value will be 0.976, and S_o will be 93.8 %. However, from the test, as the specimen gets fully saturated, back calculations gives $e_o = 0.995$ and $S_o = 92$ %.

From curve :

$$C_c = \frac{0.958 - 0.834}{\log \frac{8}{1}} = 0.1$$

$$m_v = \frac{0.958 - 0.923}{1 + 0.958} \times \frac{1}{1 - 0.5}$$

$$= 0.036 \text{ cm}^2/\text{kg}$$

(2) In a consolidation test with double drainage, the stress increment was from 1 to 2 kg/cm^2, the initial height of specimen was 19 mm and the dial reading at zero loading was zero. Calculate C_v using both Taylor's and Casagrande's methods.

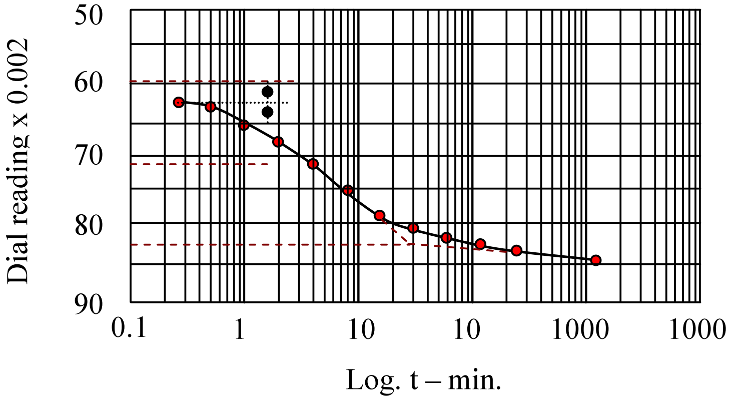
Date	Time	Elapsed Time Min.	Dial reading x 0.002 mm
21/2/90	10 : 00	0	552
		0.25	625
		0.5	635
		1	652
		2	678
		4	712
		8	752
	10 : 15	15	783
	10 : 30	30	802
	11 : 00	60	817
	12 : 00	120	825
	14 : 00	240	832
22/2/90	10 : 00	1440	846

Solution:

Casagrande's method :

$$0.197 = \frac{C_V \times 3.9}{(\frac{1.7577}{2})^2}$$

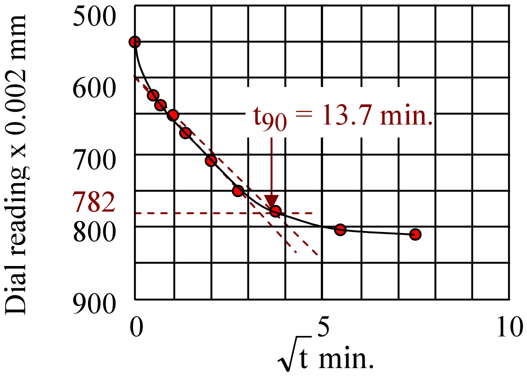
$$\therefore C_V = 0.035 \text{ cm}^2/\text{min}.$$



Taylor's method :

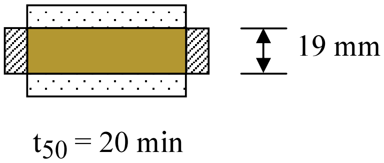
$$0.848 = \frac{C_V \times 13.7}{(\frac{1.7436}{2})^2}$$

$$\therefore C_V = 0.047 \text{ cm}^2/\text{min}.$$



(3) A consolidation test was carried out on an undisturbed clay sample 19 mm thick, extracted from a clay layer 4 m thick. Under a certain stress increment, the sample reached 50 % consolidation in 20 min, when allowing double drainage. Find the time for 50 % consolidation in the field under the same stress increment assuming double, or single drainage.

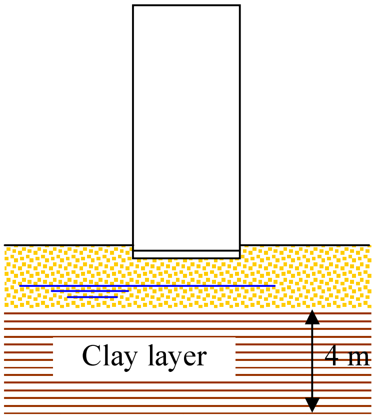
Solution:



$$T_V = \frac{C_V \cdot t}{d^2}$$

$$0.197 = \frac{C_V \times (20)}{(\frac{1.9}{2})^2}$$

$$C_V = 0.0089 \ \text{cm}^2/\text{min}$$



(a) For double drainage in the field :

$$0.197 = \frac{0.0089 \times t}{(\frac{400}{2})^2}$$

$$t = 1.68 \ \text{year}$$

(a) For single drainage in the field :

$$0.197 = \frac{0.0089 \times t}{(400)^2}$$

$$t = 6.74 \ \text{year}$$

(4) Two samples (A) and (B) were extracted from two clay layers. A specimen was prepared from each sample and tested in the oedometer. The specimens were 2 and 3 cm thick, respectively. Under a stress range of 2 to 4 kg/cm^2, the void ratios for specimens (A) and (B) change from 0.68 to 0.5 and from 0.72 to 0.62, respectively. Furthermore, the time required to reach 50 % consolidation for specimen (A) was one quarter of that for specimen (B). Find the ratio between the permeabilities of the two clay samples within the prementioned stress range.

Solution:

For specimen (A):

$$0.197 = \frac{C_{v(A)} \times \frac{t}{4}}{(\frac{2}{2})^2}$$

$$\therefore\ C_{v(A)} = \frac{0.788}{t}\ \text{cm}^2/\text{ unit time}$$

$$m_{v(A)} = \frac{0.68 - 0.5}{1 + 0.68} \cdot \frac{1}{4 - 2} = 0.0536\ \text{cm}^2/\text{kg}$$

For specimen (B):

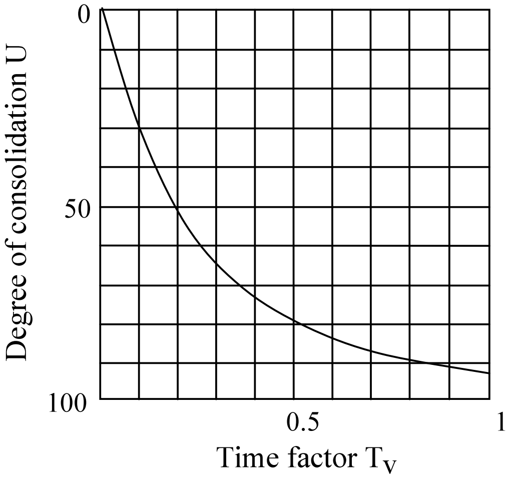
$$0.197 = \frac{C_{v(B)} \times t}{(\frac{3}{2})^2}$$

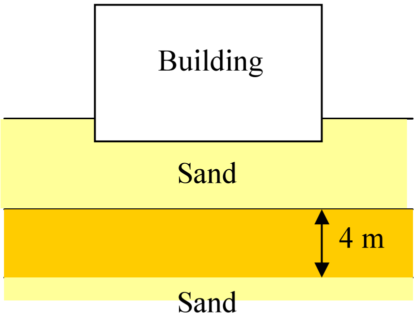
$$\therefore\ C_{v(B)} = \frac{0.443}{t}\ \text{cm}^2/\text{ unit time}$$

$$m_{v(B)} = \frac{0.72 - 0.62}{1 + 0.72} \cdot \frac{1}{4 - 2} = 0.0291\ \text{cm}^2/\text{kg}$$

$$\therefore \frac{k_{(A)}}{k_{(B)}} = \frac{C_{v(A)} \cdot m_{v(A)} \cdot \gamma_w}{C_{v(B)} \cdot m_{v(B)} \cdot \gamma_w} = \frac{\frac{0.788}{t}(0.0536)}{\frac{0.443}{t}(0.0291)} = 3.28$$

(5) The shown building is assumed to settle ultimately 12 cm. Measurements indicated that it settled 3 cm in the first month. How long would it take to reach 50 % settlement. What will be the settlement after 1 year.





Solution:

For t = 1 month :

$$U = \frac{3}{12} = 25\,\% \qquad \text{From curve } T_V = 0.06$$

$$0.06 = \frac{C_V(\frac{1}{12})}{(\frac{4}{2})^2} \qquad \therefore C_V = 2.88 \text{ m2/year}$$

To find t_{50} :

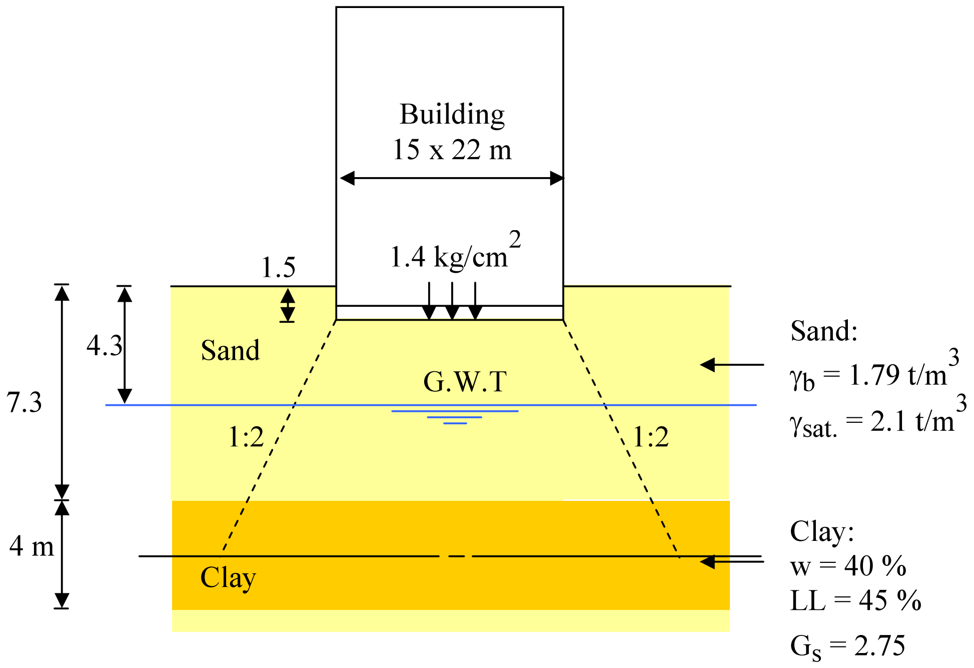
$$0.197 = \frac{2.88 \times t_{50}}{\left(\frac{4}{2}\right)^2} \qquad \therefore t_{50} = 3.28 \text{ month}$$

To find settlement after 1 year :

$$T_V = \frac{2.88 \times 1}{\left(\frac{4}{2}\right)^2} = 0.72 \qquad \therefore U = 86\,\%$$

$$\therefore\ s = 12 \times 0.86 = 10.3 \text{ cm}$$

(6) For the shown building and soil formation, calculate the average settlement of the building due consolidation of the clay layer. Use the approximate method for stress distribution in soil.



Solution:

To calculate the overbyrden stress at C.L. of the clay layer:

$$e_{o(clay)} = 2.75 \times 0.4 = 1.1$$

$$\sigma'(clay) = \frac{2.75 - 1}{1 + 1.1} \times 1 = 0.85\ t/m^3$$

$$\therefore \sigma_o' = 1.79 \times 4.3 + 1.1 \times 3 + 0.85 \times 2 = 12.657\ t/m^2$$

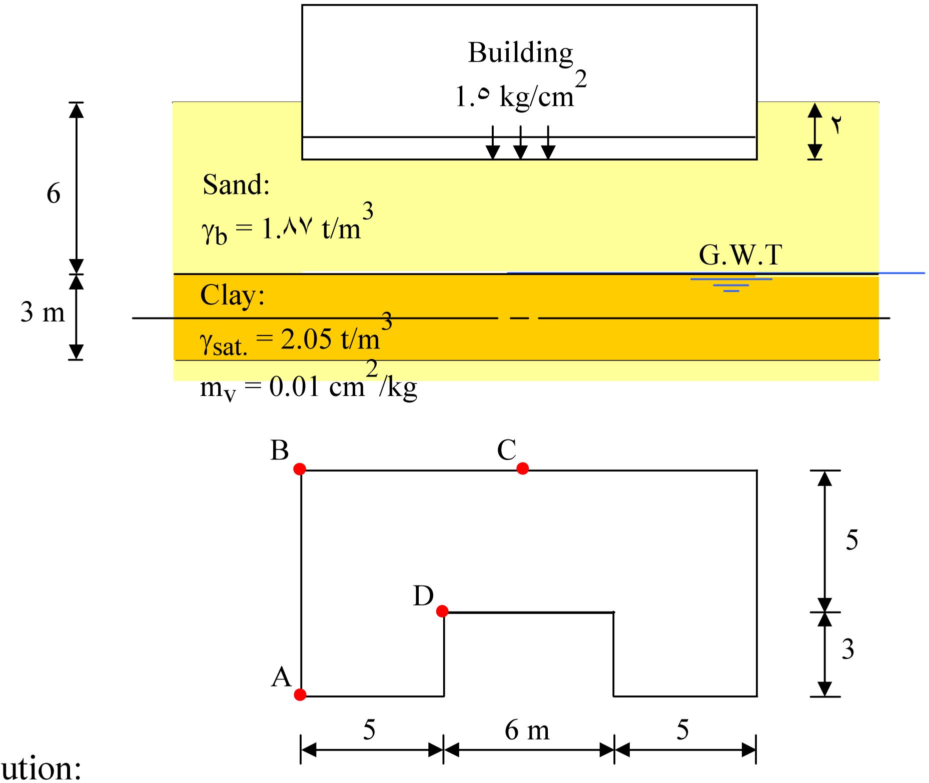
The average stress increase at C.L. of the clay layer due to the building is calculated following the approximate method.

$$\therefore \Delta\sigma' = \frac{14 \times 15 \times 22}{(15 + 7.8)(22 + 7.8)} = 6.8\ t/m^2$$

$$C_c = 0.009(45 - 10) = 0.315$$

$$\therefore s = \frac{0.315}{1 + 1.1} \times 400 \times \log\frac{12.657 + 6.8}{12.657} = 11.2\ cm$$

(7) Calculate the settlement of points A, B, C and D at the foundation of the shown building due to consolidation of the clay layer. Assume that the foundation is ideally flexible.



Solution:

Find the stress increase ($\Delta\sigma$) at the C.L. of the clay layer under points A, B, C and D by using the Newmark chart.

$z = 4 + 1.5 = 5.5$ m

Draw plan for the building to scale: line AB = 5.5 m. From the Newmark chart find N for each point. Calculate the stress increase:

$\Delta\sigma = 0.005 \times 1.5 \times N$.

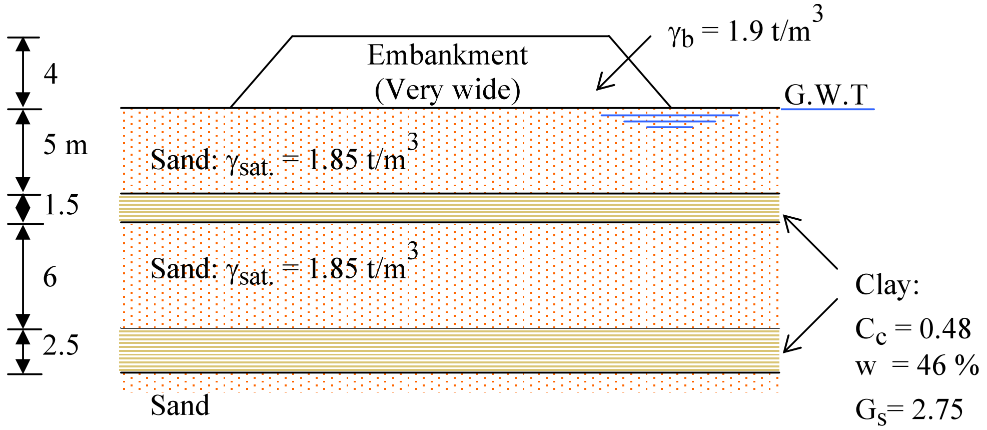
Then calculate the settlement for each point from the relation:

$S = 0.01 (\Delta\sigma) 300$.

Results are tabulated as follows:

Point	A	B	C	D
N	39	43	76	84
$\Delta\sigma$ – kg/cm^2	0.29	0.32	0.57	0.63
s - cm	0.88	0.97	1.71	1.89

(8) The stratification at a site contains two clay layers, as shown in figure. It is required to construct an embankment of large width. For the given data, calculate the settlement of the embankment.



Solution:

$$\Delta\sigma = 1.9 \times 4 = 7.6 \text{ t/m}^2$$

$$e_{o\,(clay)} = 2.75 \times 0.46 = 1.265$$

$$\gamma'(\text{clay}) = \frac{2.75-1}{1+1.265} \times 1 = 0.77 \text{ t/m}^3$$

For upper clay layer:

$$\sigma_o' = 0.85 \times 5 + 0.77 \times 0.75 = 4.829 \text{ t/m}^2$$

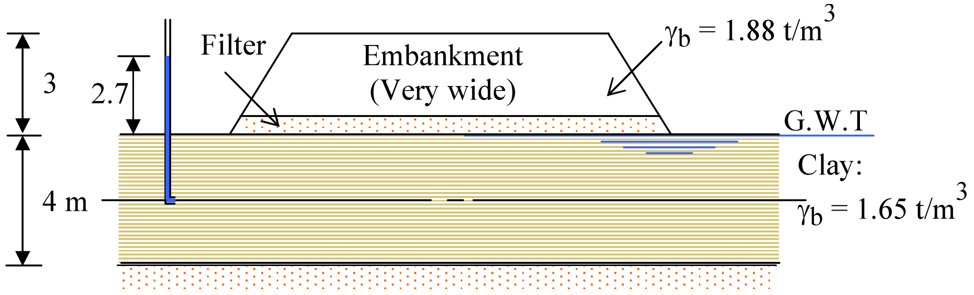
$$s_1 = \frac{0.48}{1+1.265} \times 150 \times \log \frac{4.829+7.6}{4.829} = 13.05 \text{ cm}$$

For lower clay layer:

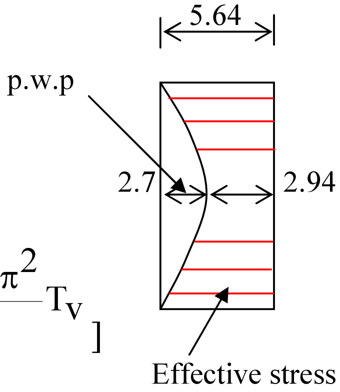
$$\sigma_o' = 0.85 \times 5 + 0.77 \times 1.5 + 6 \times 0.85 + 0.77 \times 1.25 = 11.47 \text{ t/m}^2$$

$$s_2 = \frac{0.48}{1+1.265} \times 250 \times \log \frac{11.47+7.6}{11.47} = 24.74 \text{ cm}$$

(9) A clay layer is consolidating in the field under the load of a temporary embankment. After 6 months a piezometer whose tip is at the centre line of the clay layer showed a pore water pressure of 2.7 t/m^2. For the given data determine the coefficient of consolidation of the clay. Also, determine the time which the clay layer will take for 90 % consolidation.



Solution:



$\Delta\sigma' = 1.88 \times 3 = 5.64 \text{ t/m}^2$

Use Eq.:

$$u_z = \frac{4}{\pi}\Delta\sigma' \sum_{N=0}^{N=\infty} \left[\frac{1}{2N+1} \sin\left\{\frac{(2N+1)\pi z}{2H}\right\} e^{\frac{-(2N+1)^2\pi^2}{4} T_v}\right]$$

Put: $u = 2.7 \text{ t/m}^2$, $\Delta\sigma' = 5.64 \text{ t/m}^2$, $z = 2$ m :

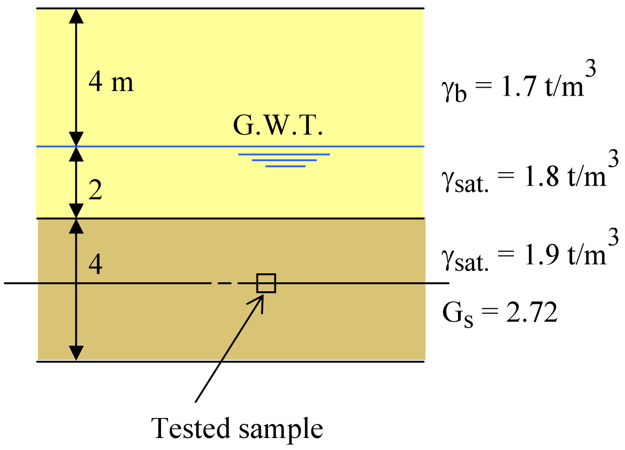
$$\therefore\ 2.7 = \frac{4 \times 5.64}{\pi} \sum_{N=0}^{N=\infty} \left[\frac{1}{2N+1} \sin\left\{\frac{(2N+1)\pi}{2}\right\} \times\ e^{\frac{-(2N+1)^2\pi^2}{4} T_v}\right]$$

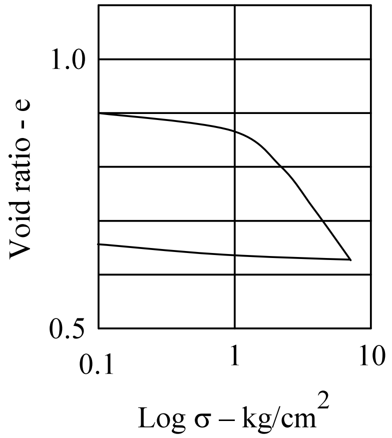
$$\&\ T_v = \frac{C_v \times t}{d^2} = \frac{Cv \times (\frac{6}{12})}{(\frac{4}{2})^2} = 0.125\, C_v$$

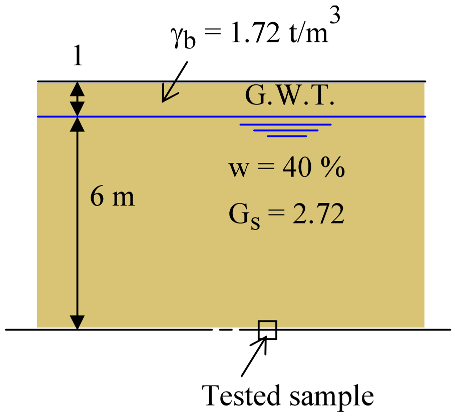
Assume different values of C_v, calculate T_v and substitute in the equation above. By trial a value of $C_v = 3.17$ m^2/year is obtained.

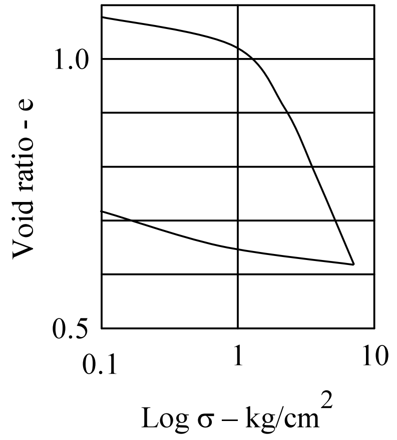
Time for 90 % consolidation: $0.848 = \frac{3.17 \times t_{90}}{(\frac{4}{2})^2}$ $\therefore t_{90} = 1.07$ year

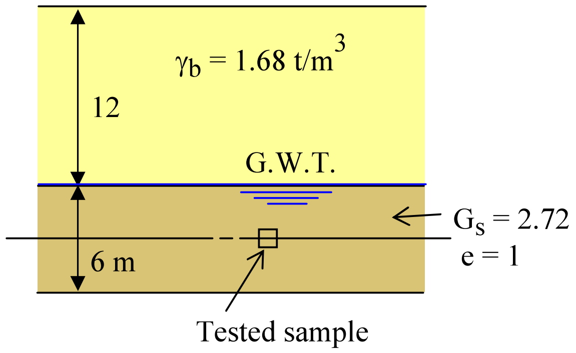
(10) The shown are three soil formations at three different sites. Results of consolidation tests on samples from the three layers at these sites are given. For each clay layer discuss the stress history and construct the field consolidation relationship.

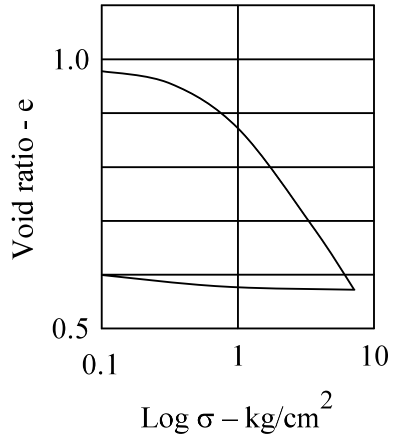












(a) $\sigma_o = 1.7 \times 4 + 0.8 \times 2 + 0.9 \times 2$

$= 10.2\ \text{t/m}^2$

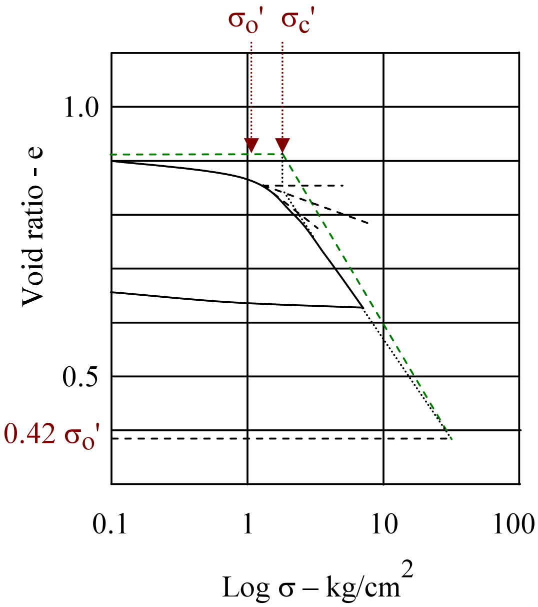
$1.9 = \dfrac{2.72 + e_o}{1 + e_o} \times 1$

$e_o = 0.91$

$0.42\, e_o = 0.383$

From curve: Clay is over consolidated

Over-consolidation ratio $= \dfrac{3}{1.02} = 2.94$



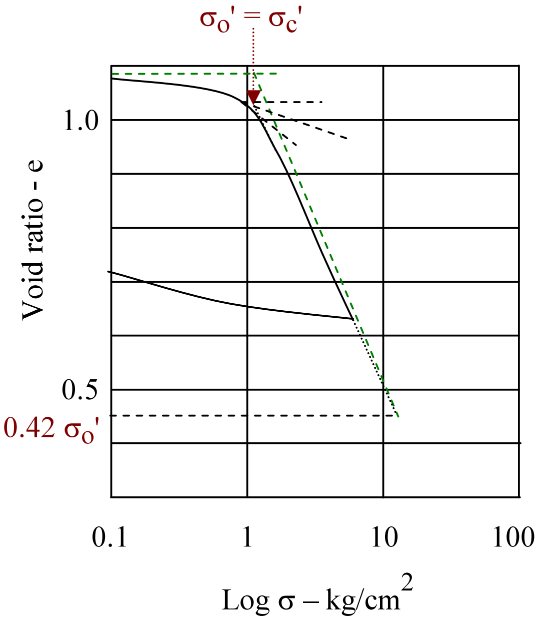
(b) $e_o = 2.7 \times 0.4 = 1.08$

$\gamma_{sub.} = \dfrac{2.7 - 1}{1 + 1.08} \times 1 = 0.817$

$\sigma_o = 1.72 \times 1 + 0.817 \times 6 = 12.6\ \text{t/m}^2$

$0.42\, e_o = 0.45$

From curve: Clay is normally consolidated



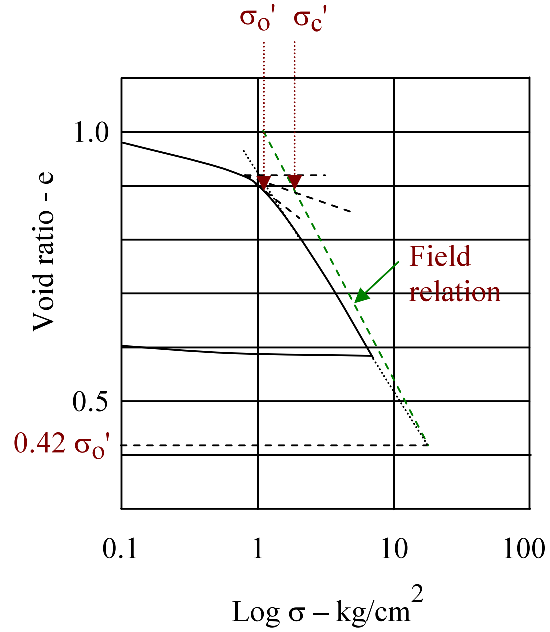
(c) $\gamma_{sub.} = \frac{2.66-1}{1+1} \times 1 = 0.83$

$\sigma_o = 1.65 \times 12 + 0.83 \times 3 = 22.29 \text{ t/m}^2$

$0.42\ e_o = 0.42$

From curve: Clay is

under consolidated



9.15 Problems

(1) The following data were obtained from a consolidation test. Draw the e-log σ relationship. Determine the compression and expansion indices. Determine also the values of the coefficient of compressibility and coefficient of volume compressibiiity for stress range 1 to 2 kg/cm^2.

σ – kg/cm^2	e
0.1	1.025
0.25	1.008
0.5	0.981
1	0.918
2	0.837
4	0.750
8	0.674
2	0.692
0.5	0.728
0.1	0.775

(2) Given the results of a consolidatlon test carried out on a soil specimen. The initial specimen height was 20 mm, and the final water content was 18.16 %. Draw the e-log σ' relation. Find C_c, and m_v for stress range 1 to 2 kg/cm^2. Take G_s = 2.7.

σ' – kg/cm^2	R x 0.002 mm
0	0
0.1	752
0.25	835
0.5	944
1	1075
2	1413
4	1961
8	2613
2	2482
0.5	2371
0.1	2164
0	2052

(3) In a consolidation test with double drainage, the stress increment was 1 to 2 kg/cm^2, the initial height of specimen was 20 mm and the dial reading at zero load was 0. From the given data, calculate the coefficient of consolidation using both Taylor's and Casagrande's methods.

Date	Time	Elapsed Time min.	Dial reading x 0.002 mm
15/9/96	8 : 00	0	460
		0.25	586
		0.5	596
		1	601
		2	610
		4	624
		8	650
		15	659
		30	682
	9 : 00	60	705
	10 : 00	120	724
	12 : 00	240	740
	16 : 00	480	752
16/9/96	8 : 00	1440	760

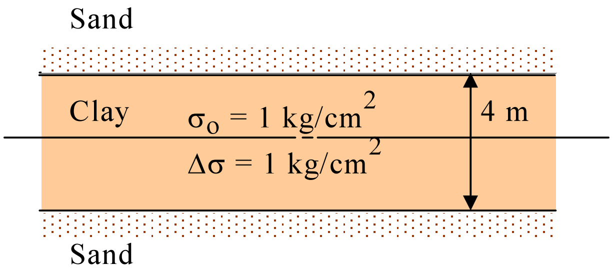
(4) A building is constructed at a site where the underlying strata includes a clay layer 3 m thick. The coefficient of consolidation of the clay was found to be 1.78×10^{-4} sec^2/sec. Find the time for the building to reach 50 % of its settlement due to consolidation of the clay.

(5) A consolidation test was carried out to investigate the probable settlement for a building causing stress increase from 1 to 2 kg/cm^2 at mid height of a clay layer 4 m thick.

Data:

Initial height of specimen = 2 cm
Change of specimen height under 1 kg/cm^2 = 0.2 cm
Change of specimen height under 2 kg/cm^2 = 0.236 cm
Time for 50% consolidation for 1 to 2 kg/cm^2 = 20 min
Sample has double drainage.

Calculate:



(a) The coefficient of volume compressibility for stress change from 1 to 2 kg/cm^2.
(b) Ultimate settlement of the building due to consolidation of the clay layer.
(c) Time for the building to reach 50 % and 90 % settlement.

(6) The following data were obtained from a consolidation test :

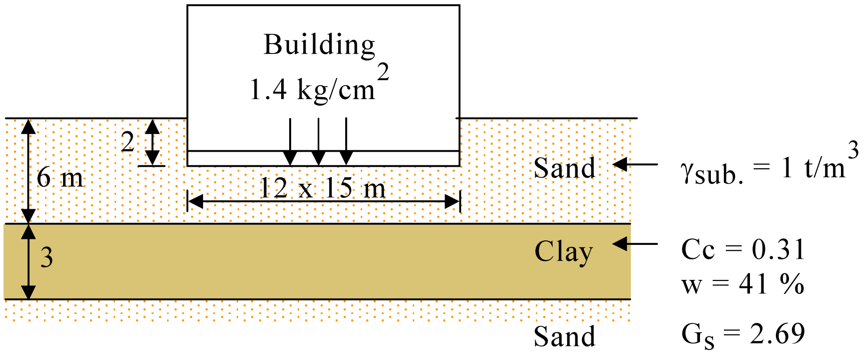
Initial height of specimen	= 2	cm
Height after consolidation under 2 kg/cm^2	= 1.782	cm
Height after unloading	= 1.761	cm
Final water content	= 34.6	%
Specific gravity	= 2.7	

(a) Find the void ratio before and after consolidation.
(b) Find m_v for stress change from 0 to 2 kg/cm^2.

(7) A saturated specimen of clay is 17.44 mm high, and its water content is 31.2 % under a consolidation stress or 1 kg/cm^2. Increase of the consolidation stress to 4 kg/cm^2 causes decrease of the specimen height by 1.81 mm. Determine the compression index for the clay. Take Gs = 2.7.

(8) Two clay layers A and B are 4 and 7 m thick, respectively. The time for 50 % consolidation for layer A is 3 months. Calculate the time for 50 % consolidation for layer B knowing that the coefficient of consolidation of layer A is twice that of layer B. Assume both layers have double drainage.

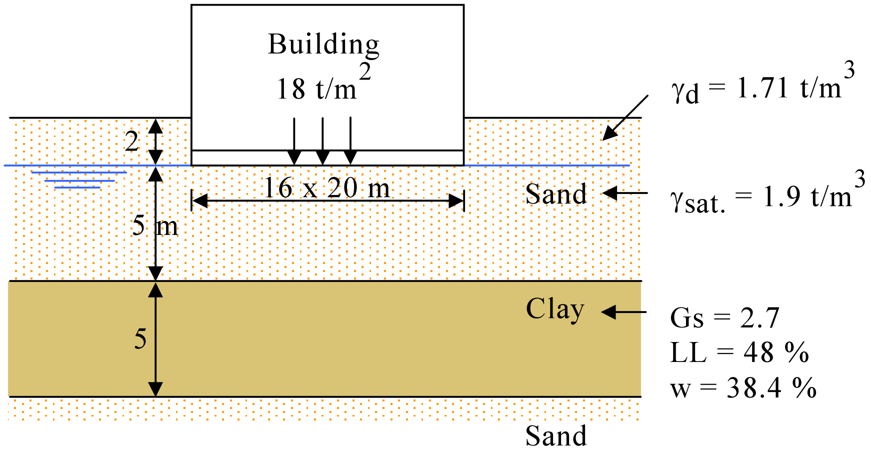
(9) For the shown soil conditions, it was found that the building settled 3 cm in 2 months. Find the ultimate settlement of the building. Find the time for 90 % settlement.



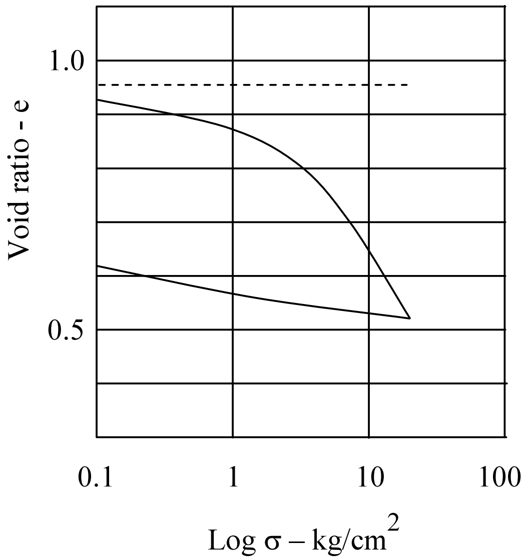
(10) In a consolidation test the void ratio decreased from 0.92 to 0.774 under a stress increase from 2 to 4 kg/cm^2. The coefficient of permeability for the given stress range is 9.4 x 10^{-9} cm/sec. Calculate the coefficient of consolidation in m^2/year.

(11) A specimen 2 cm high reached 40 % consolidation in 20 min under single drainage condition. How long the clay layer from which the specimen is obtained will reach the same degree of consolidation, if it is 3.3 m thick and drains from both sides.

(12) For the shown soil formation, calculate the average settlement of the shown building.



(13) For the shown e-log σ' relationship obtained from a consolidation test, find the compression index C_c, and the pre-consolidation stress σ_c'. If the overburden stress is 1.20 kg/cm^2, and the field void ratio is 0.961, comment on the stress history of the tested soil. Construct the field consolidation relation and find C_c(field).



(14) A clay layer 2.3 m thick is consolidating under an average stress increase of 1.1 to 2.2 kg/cm^2. Find the pore water pressure and the effective stress for 70 % consolidation occurring at the centre line of the layer. Assume double drainage.

(15) For a consolidating clay layer 8 m thick, drained from both sides, the initial void ratio is 0.94, the coefficient of compressibility is 0.042 cm^2/kg, and the coefficient of permeability is 9.4 x 10^{-9} cm/sec. Calculate the coefficient of consolidation and the time to reach 50 % and 90 % consolidation.

* * *